PHYSICS QUALIFYING EXAM UCI September 23, 1974

CLASSICAL MECHANICS

(Do 4 of these problems) Two Hours

1. In search of more living space, an intelligent race plans to redistribute their planet's mass into a giant ring, centered on their star and spinning with angular frequency Ω. Their leading physicist argues that this design is stable. Evaluate this claim. (Drawing pictures is usually a clearer method of explanation.)

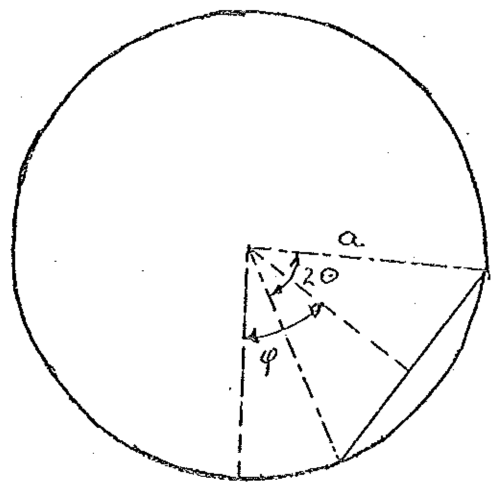
2. A cylinder of radius R is filled to a height $\mathcal{J}_o$ with water. Then it is set spinning with angular frequency Ω. Find the curve described by the surface of the water, and the height $\mathcal{J}$ of the water at the axis of the cylinder.

3. a) In the early 1960's physicists at UCSD proposed a cable be lowered from a synchronous satellite (whose orbital period equaled 24 hours, in the plane of the equator) all the way to the surface. Then packages could be put in orbit simply by running them up the cable on a pulley system, or something similar. Find the thickness of the cable as a function of height.

   b) We dig a straight tunnel through the earth at an angle less than 90^o to vertical, say between London and Moscow. A train then slides without friction back and forth between the cities. If the train moves freely, find the period of its motion.

4. A particle of mass m moves in a one-dimensional harmonic potential well $V(x) = \frac{1}{2} kx^2$. The parameter k varies slowly with time. What is the adiabatic invariant I(k) for this system?

5. A uniform rod in a uniform graviational field slides with its ends on a smooth vertical circle of radius a as shown in the figure. If the rod subtends an angle of $2\theta < 180^{\circ}$ at the center of the circle, find the frequency of small oscillations of the rod (small φ).



6. Which of the following coordinate transformations is canonical?

a) $Q = q^{\frac{1}{2}}\cos 2p \qquad P = q^{\frac{1}{2}}\sin 2p$

b) $Q = -p^2, \qquad P = -q^2$

c) $Q = \ln(p^2 + q^2), \quad P = \tan^{-1}\frac{q}{p}$

QUANTUM MECHANICS II

(Do 2 of these 4 problems. One hour)

1. A particle moves above a table (surface in the xy plane) in the gravitational potential mgz. Through use of the uncertainty principle, estimate the quantum mechanical ground state energy of the particle, and its average distance above the table. Estimate this average distance for an electron.

2. a) What are the spectroscopic terms (LSJ) of the lower energy states of the sodium atom (Z = 11).
   b) For a positive spin orbit coefficient, calculate the relative fine structure splitting and sketch the energy level diagram.

3. Describe in general physical terms under what conditions the following approximation methods for scattering would be used, and what throwing away higher order terms will neglect.
   a) Born Approximation
   b) Phase Shifts and Partial Waves
   c) Resonance Approximation
   d) Eikonal Approximation
   e) WKB

4. Derive the expression for the Hamiltonian in the presence of a weak uniform magnetic field to leading order.

Physics Qualifying Examination UCI September 23, 1974

QUANTUM MECHANICS I

(Do 4 of these 5 problems. 3 hours.)

1. An electron moves in a crossed, (D.C.) static electric and magnetic field. Find the quantum mechanical energy levels of the electron. Take the electric field along the z axis, and the magnetic field along the x axis.

2. Given the following wave function,

$$\psi(r,\theta,\varphi) \propto r^3 e^{-Cr} \cos\theta \sin^2\theta e^{-2i\varphi}$$

Calculate:

a) The z component of angular momentum.

b) The total angular momentum.

c) The potential, $V(r)$.

d) The energy if $V(r) \underset{r \to \infty}{\longrightarrow} 0$

3. An irreducible tensor operator $T(k,q)$ under the rotation group can be coupled with a representation of the group to form a new representation

$$|j_1 j_2 jm\rangle' = \sum_{m_1 m_2} T(j_1 m_1)|j_2 m_2\rangle\langle j_1 j_2 m_1 m_2|j_1 j_2 jm\rangle \quad .$$

a) As a part of verifying that this new state is a representation, evaluate

$$J_z|j_1 j_2 jm\rangle' .$$

b) Using the identity $J_- J_+ = \vec{J}^2 - J_z^2 - \hbar J_z$

show that the norm of these states $'\langle jm|jm\rangle'$ is independent of m.

4. Consider the ground state of the one dimensional harmonic oscillator with the Hamiltonian

$$H = \frac{\bar{p}^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda_o x^4$$

Use a variational method with the variable parameter α to bound the ground state energy.

Given:
$$\int_{-\infty}^{\infty} x^{2n} e^{-\alpha^2 x^2} = \begin{cases} \dfrac{\sqrt{\pi}}{\alpha} & n = 0 \\ \dfrac{1(3)(5)\cdots(2n-1)\sqrt{\pi}}{2^n \alpha^{2n+1}} & n = 1, 2, \cdots \end{cases}$$

5. A spin one particle is initially in the state $S_z = +\hbar$

a) What are the possible results of a measurement of S_x?

b) With what probability would they occur?

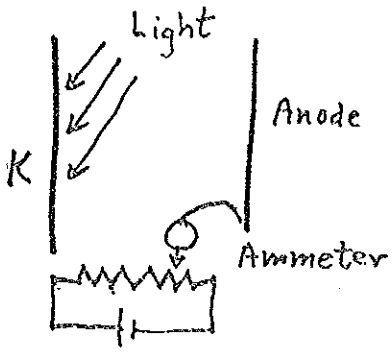
c) What is the expectation value of S_x?

d) If the measurement of S_x gives $S_x = +\hbar$, what would a successive measure of S_x give? Of S_z?

GENERAL PHYSICS

(Do 10 of 18 problems) Two Hours

1. A surface of metallic potassium is illuminated with monochromatic of various wavelengths. In each case, the potential difference is found for which no electrons reach the anode. Two pieces of data are given:



Wavelength (Å)	2000	5000
Potential difference-volts	f.11	0.41

Find the value of Planck's constant.

2. The "saturation" field that can be obtained from magnetizing iron is about 20,000 gauss. Assuming that this field is produced by the alignment of a few electron spins per atom, estimate the spin-magnetic moment of the electron.

3. The flux of energy from the sun at the earth is about 2 calories per square centimeter per minute on an area perpendicular to the direction of incidence. Under the assumption (a very poor one, to be sure) that the incident radiation is monochromatic, estimate the amplitude of both B and E.

4. Hydrogen in the ground state is in a $2s_{1/2}$ state. The spin of the proton is 1/2 and the hyperfine structure splitting is about 1420 Mh_z. The proton magnetic dipole moment is about 2.8 nuclear magnetons. Estimate the effective magnetic field within the atom at the point where the proton exists.

5. Making use of the equivalence principle (gravitation equivalent to accelerated frame) derive an expression for the gravitational red shift. Is this measurable near the earths surface?

6. Hydrogen gas in a laboratory discharge tube shows a discrete spectrum. At the same temperature and pressure, but in a massive system such as a star, it shows more nearly a black body radiation spectrum. Explain.

7. Can a spin-$\frac{1}{2}$ system have a permanent electric dipole moment if

   (a) Parity is conserved?

   (b) Parity is not conserved but time reversal invariance holds?

   Why?

8. Can a gamma ray convert into an electron-positron pair in (i) free space (ii) an aluminum plate (iii) a lead plate? In which medium is the mean free path for conversion shorter?

9. The decay of a ρ^o meson (spin-parity = 1^-)

$$\rho^o \to \pi^+\pi^- \text{ is allowed, but}$$

$$\rho^o \to \pi^o\pi^o \text{ is not}$$

   Why?

10. Can a Σ^+ hyperon (spin-parity $\frac{1}{2}^+$) have a static electric quadrupole moment? Why.

11. If a medium has a non-linear characteristic such that the velocity of propagation of a pulse increases with the amplitude of the pulse, what will happen to a disturbance for which the amplitude as a function of position is bell shaped initially? Draw a picture.

12. Energy is delivered to a clear 150 watt incandescent bulb. Immediately, about a foot away, your hand feels the heat. Now quickly touch the lamp. It still feels cold. Explain this. Explain how the hand is heated. Why is the bulb gas filled and not highly evacuated?

13. X-rays exhibit all the phenomena associated with electromagnetic waves-reflection, refraction, polarization, diffraction, interference. How would you undertake to show these phenomena with x-rays?

14. Snap your finger. Notice that one finger quickly moves from being against your thumb to contact with the palm. Keeping the other fingers extended (and not curled against the palm), snap your finger again. Explain the difference in the sound in the two cases.

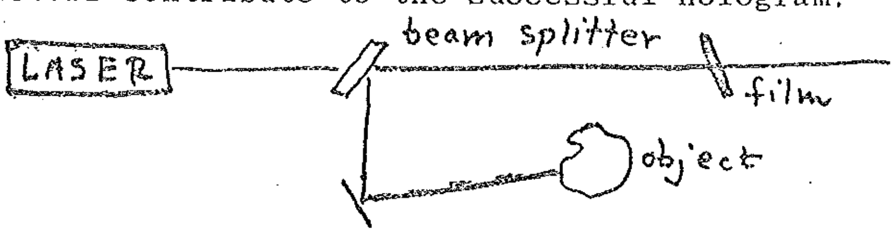
15. a) Define the term <u>isotope</u>

b) In the past few decades, an enormous amount of research on methods of separating isotopes has been done. Explain briefly why it is difficult to separate isotopes.

c) Name one technique used in practice for separating isotopes.

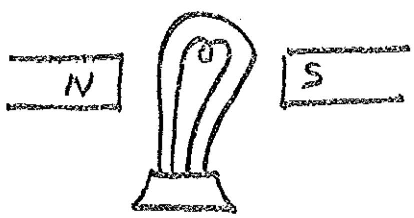
d) Why is there such intense interest in the development of efficient methods for isotope separation?

16. In holography making, the object to film distance is normally a meter or less. Because phase and not intensity are being recorded, how do the important geometrical factors contribute to the successful hologram?



17. Explain what data you would need to unambiguously verify to the layman that the earth rotates.

18. Two identical clear-glass, incandescent, carbon-filament lamps are energized, one on AC current and the other on DC current.



If now a strong horseshoe magnet is brought around each lamp, what happens to the filament in each case?

Physics Qualifying Exam UCI September 25, 1974

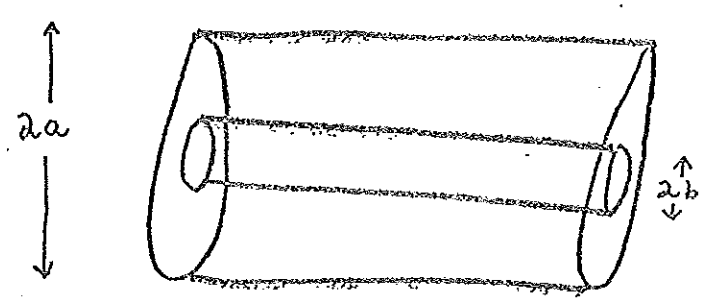
ELECTRICITY AND MAGNETISM

(Do 4 of the 6 problems) Two Hours

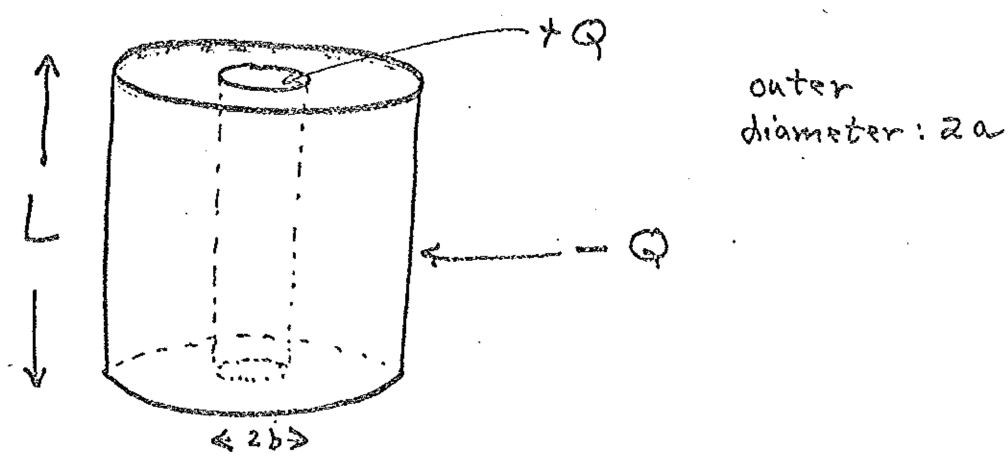
1. A square of copper is released near the earth's surface. A constant magnetic field B is parallel to the earth's surface and normal to the plane of the square at all times. The square has side length L, constant resistance R and mass m. Gravitational acceleration g is constant. Find the square's vertical velocity v as a function of time.

2. Plasma effects occur both in conventional plasmas and solids. Consider propagation of sound in a metal of n atoms per unit volume, assuming the conduction electrons are free and degenerate with Fermi temperature T_F. Find the velocity of sound in this metal.

3. A coaxial cavity with perfectly conducting walls and end plates is excited in its lowest mode. In terms of W, the energy stored in the electromagnetic field, determine the time-averaged pressure exerted by the field on the side walls and end walls.

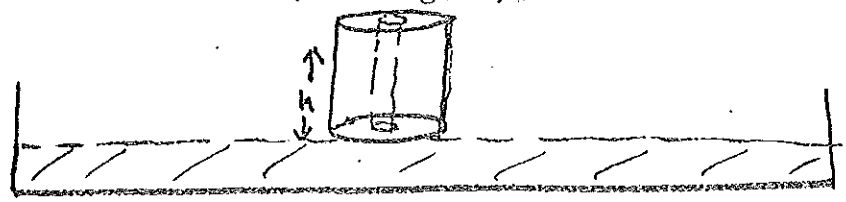


4. Consider a cylindrical capacitor which has been charged and then disconnnected from the voltage source. (see Fig. 1).



4. (Continued)

This capacitor is put into a large dish containing a liquid of dielectric constant K. The liquid rises up into the space between the plates of the capacitor until an equilibrium height is reached. (see Fig. 2).



(a) Derive an equation which could be solved to find the equilibrium height (h) of the liquid, Neglect capilarry action. From the solution of this equation it can be shown that the increase in gravitational potential energy at equilibrium is less than the decrease in electrical potential. Explain without calculation the physical origin of this difference.

(b) If the voltage source remains connected when the capacitor is put into the dish give the physical basis, without calculation, for the changes from part (a) required in the analysis.

5. An anisotropic, non-magnetic medium is placed in a large electrostatic field E_o acting in the z-direction. This makes the dielectric tensor anisotropic:

5. (Continued)

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix}, \quad \varepsilon_1 \neq \varepsilon_2$$

a) Obtain the general real solution for a wave propagating through this, now anisotropic, medium in the x-direction.

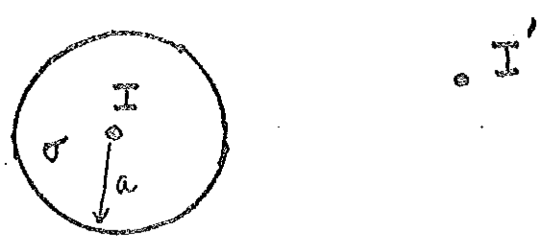
b) A linearly polarized wave of frequency ω and with polarization vector $\xi = (0, 1/\sqrt{2}, 1/\sqrt{2})$ is incident at $x = 0$. Find an expression for the distance x at which the polarization vector will be at right angles to its original direction.

6. Consider this situation, with long (effectively infinite) currents I and I′ interacting with a cylindrical conductor of radius a and conductivity σ. I and I′ are parallel. I is somehow fixed at the center of the conductor and doesn't move. I′ can move, and is a distance r from I ($r > a$).

a) Find the force F on I′ as a function of r. Consider σ infinite. Explore F(r) as a function I/I′.

b) Find the energy of the system as a function of r, with σ infinite.

c) If the answer to (b) can be infinite, for some r, what physical fact removes the effect? Estimate the maximum energy.



MATHEMATICAL PHYSICS

(Do 4 of these 5 problems) Two Hours

1. Problem:

Evaluate
$$\int_o^\infty \frac{\sqrt{x}}{1+x^2}\,dx \quad .$$

Show your path of integration in the complex plane.

2. Solve the equation

$$\frac{dy}{dx} + y = xe^{-ax} + c$$

where c is a constant and y(o) = 1.

3. Consider a perfectly flexible homogeneous string of linear mass density ρ and length L, rotating with constant angular velocity Ω around a vertical axis through one end of the strong.

a) Neglecting gravity, use Hamilton's principle to show that the equation describing small transverse vibrations of the string (in a plane through the axis of rotation, rotating together with the string) is

$$\frac{\partial}{\partial x}\left((L^2 - x^2)\frac{\partial u}{\partial x}\right) - \frac{2}{\Omega^2}\frac{\partial^2 u}{\partial t^2} = 0 \quad . \qquad (1)$$

where u(x,t) denotes the deviation from the unperturbed position at distance x from the axis of rotation and time t. Show that the appropriate boundary conditions are

$$\begin{aligned} u(o,t) &= 0 \\ u(L,t) &= \text{finite} \quad . \end{aligned} \qquad (2)$$

b) Use the method of separation of variables to find the normal modes of the string.

c) Find the solution corresponding to the following initial conditions

$$\begin{aligned} u(x,0) &= \alpha x^3 \qquad 0 \le x \le L \\ u_t(x,0) &= 0. \end{aligned}$$

4. The Bernoulli Numbers are defined by

$$\frac{x}{e^x - 1} = \frac{B_0}{0!} + \frac{B_1}{1!} x + \frac{B_2}{2!} x^2 + \cdots$$

Find

(a) A general relation between the B_n which allows you to evaluate them.

(b) Display B_0, B_1, B_2, B_3.

5. An infinite solid has temperature distribution $T(x,t)$. Nothing depends on y or z. Find $T(x,t)$ if $T(x,0) = \delta(x)$.

PHYSICS QUALIFYING EXAM UCI September 24, 1974

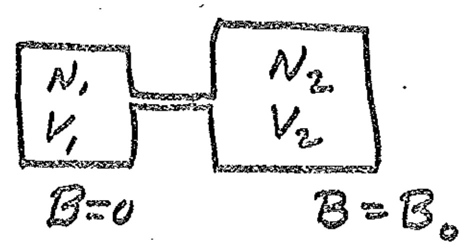
THERMODYNAMICS AND STATISTICAL MECHANICS

(Do 4 of 6 problems) Two Hours

1. A gas of N particles is contained in a cylinder of cross sectional area A, but of infinite height. The gravitational potential can be taken as mgz for all z. Each molecule is rigid (i.e., it cannot vibrate) and may rotate. Each molecule has a nondegenerate electronic ground state and a three-fold degenerate first excited state at energy ϵ_o above the ground state. (All other electronic states can be neglected.) What is the entropy of the system assuming that kT is large compared to the splitting of the translational and rotational states?

2. Consider a gas of N indistinguishable, non-interacting, diatomic molecules.
   a) Calculate the total partition function including translation, vibration, and rotation. Assume kT large compared to energy spacing of rotational and translational states, but small compared to energy spacing of vibrational states.
   b) Calculate mean energy $\bar{E}$, mean pressure $\bar{p}$, and molar specific heat at constant volume.

3. Two boxes both at temperature T containing molecules of spin $\frac{1}{2}$ and magnetic moment μ are joined by a small tube. The respective volumes of the boxes are V_1 and V_2. The molecules are free to move from one box to the other. The total number of molecules is N. There is a constant magnetic field of strength B_o in box 2 and no magnetic field in box 1.



$$N_1 + N_2 = N$$

3. (Continued)

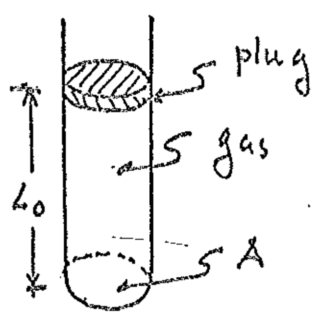
a) Calculate the partition functions for box 1 and box 2 assuming N_1 molecules in box 1 and N_2 molecules in box 2.

b) State the criterion for equilibrium between the two boxes.

c) Calculate the ratio of the number of molecules in the two boxes in equilibrium at temperature T.

4. A hot star can lose light particles easier than heavy ones. Make this argument quantitative and apply it to light electrons vs. heavy protons. How long would the solar "wind" be composed solely of electrons, and what mechanism would make the wind evolve in composition toward the charge-neutral one we have today?

5. A plug of mass M rests on top of a column of ideal gas with cross sectional area A and length L_o. The ratio of specific heats of the gas is $C_p/C_v = \gamma$.



What is the period of small adiabatic oscillations of the plug?

6. Derive the equation

a) $$\left(\frac{\partial C_v}{\partial V}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V$$

b) Prove that C_v of an ideal gas is a function of T only.